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A.S.R.E.
TECHNICAL NOTE
GX-55-1

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AN ELECTRONIC SERVO SIMULATOR

by

R. W. RENSON

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ADMIRALTY SIGNAL AND RADAR ESTABLISHMENT
PORTSDOWN, COSHAM,
PORTSMOUTH, HANTS.

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A.S.R.E. TECHNICAL NOTE GX-55-1
DATE: 14TH APRIL, 1955

AN ELECTRONIC SERVO SIMULATOR

by

R. W. RENSON

Approved by W. D. Mallinson
Head of Division

SUMMARY:

The construction of a simple electronic servo simulator is described and some of the possible applications are indicated.

Admiralty Signal and Radar Establishment
Portsdown, Cosham,
Portsmouth, Hants.

P.T.O. for Initial Distribution and References to Illustrations

FIGURE NUMBER	REF. NO. OF DRAWINGS OR PHOTOGRAPHS	SECURITY CATEGORY	FIGURE NUMBER	REF. NO. OF DRAWINGS OR PHOTOGRAPHS	SECURITY CATEGORY
1	99281	U	10	99288	U
2	99282	U	11	99290	U
3,4	99283	U	12	99291	U
5	99284	U	13	99292	U
6	99285	U	14	99293	U
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AN ELECTRONIC SERVO SIMULATOR

1. INTRODUCTION

A modified version of an electronic servo simulator, first described in a paper by Williams and Ritson, (Ref. 1). was built in the laboratory to investigate the servo parameters of an auto-tracking radar. It proved such a useful instrument that a better engineered version has been built. This note will describe the equipment and indicate some of the possible uses.

The simulator is basically a special purpose analogue computer containing a number of variable elements. These reproduce electronically the characteristics of the system to be investigated. It is emphasised that the simulator now described has been designed for the investigation of closed-loop problems. It would not be accurate enough for the different techniques used in open-loop computation.

The basic element is a high-gain D.C. amplifier which has been carefully designed to give a stable performance. By the use of selected passive networks in the input and feedback circuits of this amplifier, basic operations can be performed. The simple theory of these circuits is included as an Appendix to this note.

There are twenty-four standard blocks in the simulator with which quite complicated loops can be built up. Interconnection and feedback component selection is simple and relatively quick.

To simulate practical problems, facilities are available for introducing limits to the velocity and acceleration of the simulated output member. Backlash and resilience effects can be studied.

The most useful input has been found to be a position step, but other step-function inputs can be used, e.g. a step-function of velocity. Alternative inputs are an S.H.M. of frequency between 0.05 c.p.s. and 100 c.p.s. and an input consisting of "noise", with an approximately flat spectrum from 0.1 c.p.s. to 10 c.p.s.

A low-speed oscilloscope is included as part of the equipment, although a pen recorder is often used to produce permanent records for comparison with the "real" system at a later date.

2. THE EQUIPMENT

2.1 General

The simulator is a self-contained unit and comprises:-

(a) The twenty-four standard blocks

(b) The input circuits

(c) Limiting and backlash circuits

(d) Low-speed oscilloscope

(e) R.M.S. computer and noise filter

(f) Stabilised power supplies

(g) Valve voltmeter for setting up circuits.

The equipment is mounted in two standard five foot Post Office racks; the layout is shown in Fig. 1. Supplies of 1 amp of 24 volt D.C. and 4.5 amps of 180 volt 500 c.p.s. are required.

2.2 The Standard Block

A circuit diagram is shown in Fig. 2. The main amplifier, consisting of V1 and V2, has a nominal open-loop gain of 550. The variation in gain of twenty-four similar circuits did not exceed ± 10%. The saturation level is ± 45 volts output and the frequency response is flat within 3 dB to 7 kc/s. The response time is 1.4 milliseconds. Relay A is included for ease of setting up, i.e. any capacity in Z4 is discharged whilst the grid of V1 is earthed.

The phase lag network (R, C) is necessary for stabilisation. Since the natural frequency of the amplifier varies from chassis to chassis, the value of R must be selected.

Amplifier V3 is a unity-gain stage by means of which the polarity of the output can be reversed when required. When a capacity is used in the feedback position, it must be isolated from the following circuit, unless this has a resistive input, by V3 or a standard block coupled as a unity gain stage.

There is provision for three summing components and one feedback component. A range of the more usual values of components, selected to within ± 1% of nominal value, are kept available for the simulator.

2.3 The Input Circuits

(a) Step-functions

A circuit is shown in Fig. 3. A D.C. voltage step can be fed via the cathode-follower V1 to the step-function relay on the Low Speed Oscilloscope (see subsection 2.6). It is then fed to the simulator circuits under examination. The step-function relay can be switch operated if the oscilloscope is not being used. (The output of the Sine Wave Generator and the Noise Source are also fed via this relay.)

Other forms of step-functions, e.g. exponentials, steps of constant velocity, etc., can be obtained by feeding the D.C. voltage step to a standard block connected to give the required time-function output.

(b) Sine-Wave Generator

This is a conventional phase-shift oscillator using a four-section R.C. network. The circuit is shown in Fig. 5.

The frequency range is 0.05 c.p.s. to 100 c.p.s. The output amplitude can be varied from zero to 125 volts (peak to peak) by means of RV2. For one particular setting of RV2 the output amplitude will not vary by more than 5% over the frequency range. The output zero is adjusted by means of RV1 and SWA.

(c) Noise Source

The circuit diagram is shown in Fig. 6. Noise is generated in the thyratron V1 and heterodyned with a 3.3 kc/s square wave in V3. The output is smoothed by a low-pass filter of time constant 0.01 seconds and is amplified in V4. RV2 controls the output magnitude, which has a maximum value of 10 volts, and RV1 controls the level. When a sharp cut-off frequency is required, the output is fed to the noise filter (see subsection 2.8).

2.4 Limiting Circuit

Limit voltages of up to ± 30 volts can be set into a simulated loop by the circuit shown in Fig. 4. The output of the circuit is taken to the high impedance limit point on the standard block (See Fig. 2).

When setting up a limit voltage, the valve voltmeter must be connected between a monitor point on the limit chassis and the limit input on the standard block.

The grid base of the output cathode follower of the standard block is 0.6 volts and this is therefore the least value of voltage which can be used on the limit circuit.

2.5 Backlash Circuit

It will be seen from Fig. 7 that a positive-going input has to overcome the bias on V2A, selected by RV2, before a change occurs at the output. Similarly, for a negative-going input, the bias on V2B must be overcome.

After, say, a positive-going input has reached a maximum and is reducing, the output will not change until the condenser C has discharged through V2B. This will therefore simulate the effect of backlash between the motor and the load with starting friction, but no inertia, on the load. The result of passing a sawtooth waveform through this circuit is shown in Fig. 8.

It is necessary to isolate the input and output of this circuit by cathode followers. To maintain unity amplification in the reset loop, an amplification of 1.15 is used following the backlash circuit. Because of the input cathode follower, special care must be exercised in setting the bias of the diodes to correspond to the voltage scale of the resetting loop. RV1 and RV4 are used to cancel the grid bases of V1A and B.

2.6 Low Speed Oscilloscope

The circuit diagram is shown in Fig. 9. The principle features are:-

(a) Time base speeds selected in three ranges. Each range has preselected calibration pips which can be switched off.

Range (i)	5 seconds to 1 second, 1 second calibration pips.
Range (ii)	1 second to 0.2 second, 0.1 second calibration pips.
Range (iii)	0.2 second to 0.04 second, 0.01 second calibration pips.

(b) A step-function relay operated at a constant distance from the start of the sweep. The polarity of the step-function can be selected by a switch.

(c) The time base is normally repetitive, the time delay being 5 seconds on range (i) and 1 second on ranges (ii) and (iii). Single-stroke or external triggered operation can be used.

(d) The X and Y plates can be fed from D.C. amplifiers giving a sensitivity of up to 50 mm/volt. Facilities are available for feeding the plates direct.

(e) A reference voltage supply is included for "backing off" when the input is not about earth potential.

(f) It is worth recalling that the relative phase of two sinusoidal voltages can be measured with the oscilloscope. The voltages are fed to the X and Y plates and adjusted separately to give equal deflection. The direction of the major axis of the ellipse will give the relative phase shift. It is sometimes more convenient to compute the phase angle as $\tan^{-1} Y_2/Y_1$ where Y_2 and Y_1 are quantities shown in Fig. 9(a).

2.7 R.M.S. Computer

The circuit diagram is shown in Fig. 10. The input voltage is scaled, squared, and integrated by normal circuit techniques. The output appears as a meter reading. The circuit is normally used in conjunction with a noise input. Integration times of 10, 25, 50, and 100 seconds can be used and are measured with a stop watch. The circuit scaling is adjusted by SWA. The integrator is reset by SWB when not in use. The mean square voltage output is that indicated by M1 multiplied by a scaling factor (1, 0.5, or 0.2) selected by SWC. The accuracy of the circuit is 5%.

2.8 Noise Filter

The filter has been designed to give unit magnification, ± 1 dB, to all frequencies less than ω after which the attenuation is 30 dB/octave.

The circuit is shown in Fig. 11. The input noise is fed through three simple R.C. low-pass filters followed by a second order filter with a transfer function (see subsection 3.2) of

$$\omega^2/(p^2+2a\omega p+\omega^2)$$

The damping factor, a, of the second order is made 0.1 and the natural frequency, ω, is 50 rad/sec. The time constants of each of the R.C. filters equal $1/\omega$.

When the input has a frequency equal to ω, the output voltage from the low pass filters is -8 dB and, for increase in frequency, falls at the rate of 18 dB/octave. The second-order filter has a sharp peak of +8 dB at frequency ω after which the output falls at 12 dB/octave. The sum of these filters provides the required characteristic.

3. OPERATION OF THE SIMULATOR

3.1 General

The simulator can be set up to produce a particular transfer function or can be used to synthesise a given system from a knowledge of its components. Once the system has been set up, measurements are carried out to determine the transient response, frequency response, etc., using the equipment described in section 2 above.

3.2 To Set Up a given Transfer Function

The transfer function is the differential equation relating the output of the servo to the input. In low-power servos operating in a linear regime, a simple transfer function generally gives a very close approximation to the behaviour of the servo.

Let p be the operator d/dt, ϕ be the input quantity and θ the output. The three systems most commonly encountered have transfer functions given by:-

$$\frac{\theta}{\phi} = \frac{b^2}{p^2+2abp+b^2} \qquad \text{.........(1)}$$

$$\frac{\theta}{\phi} = \frac{2abp+b^2}{p^2+2abp+b^2} \qquad \text{.........(2)}$$

$$\frac{\theta}{\phi} = \frac{2abp^2+cb^2p+b^3}{p^3+2abp^2+cb^2p+b^3} \qquad \text{.........(3)}$$

These represent servo systems having finite velocity lag, zero velocity lag and zero acceleration lag respectively.

One method of producing each of these transfer functions is shown in Fig. 12.

The simplest system is that producing the zero velocity-lag transfer function, equation (2). From Fig. 12(b), using the simple expressions for the blocks developed in the Appendix,:-

$$\text{Let} \quad e = -(\phi+\theta)$$

$$\text{and} \quad e\left(\frac{R_3}{R_1} + \frac{1}{pC_1R_1}\right)\frac{1}{pC_2R_2} = \theta$$

$$\frac{\theta}{\phi} = -\frac{(pR_3/R_1C_2R_2)+(1/R_1C_1R_2C_2)}{p^2+(pR_3/R_1C_2R_2)+(1/R_1C_1R_2C_2)} \qquad \text{.........(4)}$$

Hence, if $R_3/R_1C_2R_2 = 2ab$

$$1/R_1C_1R_2C_2 = b^2$$

equations (2) and (4) are identical.

The system producing the finite velocity-lag transfer function, equation (1), is shown in Fig. 12(a). It will be seen that

$$e = -(\phi + \theta + \frac{R}{R_3}R_2C_2p\theta)$$

$$\text{and} \quad \theta = \frac{1}{pR_1C_1} \times \frac{1}{pR_2C_2} \times e$$

Therefore
$$\frac{\theta}{\phi} = -\frac{1/R_1C_1R_2C_2}{p^2+(pR/R_3R_1C_1)+(1/R_1C_1R_2C_2)}$$

For identity with equation (1):-

$$R/R_3R_1C_1 = 2ab$$

$$1/R_1C_1R_2C_2 = b^2$$

Figure 12(c) shows a method of producing a zero acceleration-lag transfer function, equation (3). Proceeding as before,

$$\theta = -\left[\left\{\frac{1}{pR_1C_1}\left(\frac{R_4}{R_2} + \frac{1}{pR_2C_2}\right) \times \frac{1}{pC_3R_3}\right\} + \left(\frac{R}{R_5} \times \frac{1}{pR_3C_3}\right)\right](\phi+\theta)$$

and for identity with equation (3):-

$$R/R_5R_3C_3 = 2ab$$

$$R_4/R_2R_1C_1R_3C_3 = cb^2$$

$$1/R_1C_1R_2C_2R_3C_3 = b^3$$

Two practical points should be noted at this stage:-

(a) When integrators are used in cascade, their time constants should be approximately equal. If this is not possible, the shortest time constant should occur at the end of the chain. Otherwise the short time constant integrator will reach its maximum output voltage before the resetting loop has had time to reduce the input step-function, i.e. the integrator will saturate.

As an example, a practical system was set up to simulate equation (2) with a = 1 and b = 10. Integrators of time constant one second followed by 1/100 second gave correct performance to a 10 volt step. Integrators of time constant 1/20 second followed by 1/5 second gave substantial errors. When the time constants were equal at 1/10 second, the system was again satisfactory.

(b) Great care must be exercised to ensure that the sign of the output of each block is correct. As stated in subsection 2.2 the output of each block can be selected positive or negative. Fig. 12 shows the polarity of the output required from each block in these particular examples.

3.3 Introduction of Delays and Simple Non-Linearities into the Resetting Loop

It will be apparent from Fig. 12(d) that delays and simple non-linearities existing in the resetting loop of a "real" system can be introduced directly into the simulated system. A standard block is used to provide the transfer function 1/(1+pT) where a simple exponential delay (of magnitude T seconds) occurs between the output and the resetting signal. The circuit simulating backlash was described in subsection 2.5 and can be used to simulate backlash between motor and resetter. If the output has a maximum velocity, this can be simulated by restricting the level of the voltage fed to the last integrating stage.

Delays present in the error signal, due to smoothing etc., can be introduced by placing the requisite block in the path of the error signal. This position is shown dotted in Fig. 12(d).

Examples of these systems will be described in subsection 4.3.

3.4 Synthesis of a given Electro-mechanical System

(a) Introduction

This alternative method of setting up the simulator is used when a transfer function is not known or when it is more complex than those discussed in subsection 3.2. It is generally necessary to use this method when high-power servos are being considered, the motor characteristics, starting friction and other non-linearities being important.

The method is described in Ref. 1. Briefly, the complete differential equation of the mechanical system is set up on the simulator. The resetting loop is closed and the servo is stabilised using the method intended for the "real" system. This simulation is straightforward but attention must be given to the scaling of voltage and mechanical motion. Care must also be exercised in choosing constants that do not saturate the electronic units.

(b) Motor and Load

The differential equation is

$$T = Ip^2\theta + fp\theta$$

where T is the output torque of the motor
θ is the angular displacement of the motor
I is the total inertia at the motor shaft
and $\underline{f}$ is the motor dynamic breaking coefficient

In a real system, torque is often produced by passing current through the motor field. Hence

$$T = Gke$$

where $\underline{e}$ is the input misalignment signal
$\underline{G}$ is the gain of the amplifier
and $\underline{k}$ is the conversion factor; milliamps in the field of the motor to oz in. torque.

This system is shown simulated in Fig. 13(a) where

$$R_1C_1 = f/T \quad R_2/R_3 = 1 \quad \text{and} \quad C_2R_3/R_2 = I/f$$

The input voltage to this circuit represents torque and hence can be limited to simulate the maximum torque available from the real system.

When the output torque is not a linear function of input voltage, the desired law may be obtained by a curve-fitting circuit such as those described in Refs. 2 and 3.

(c) Friction and Windage Torque

In a practical system the motor has to do work other than accelerating the load, e.g. friction must be overcome. Since the sign of friction torque is always such as to oppose the applied torque, the system shown in Fig. 13(b) is used. For small movements of the input or output an equal and opposite voltage appears in the simulator at the point equivalent to torque. The output therefore remains stationary. As the input increases, the limit circuit operates, a signal appears at e', and the output changes to null this signal.

The value of friction torque varies with speed in a manner that is not precisely known. Its value when motion just commences is considerably lower than the value at standstill and this is a major cause of jerky motion at low speed. It can be simulated by reducing the bias on the diodes by a relay operated when the output of the first integrator of Fig. 13(a) moves from zero.

Another source of torque which must be overcome by the motor is that due to windage. This will be appreciable on a large aerial and can easily be simulated. It must be scaled back to the motor, converted into voltage units and subtracted from the signal e of Fig. 13(a). This torque will probably vary as the output position or velocity changes. These functions occur as voltages in the simulator and can therefore be used to modify the simulated windage torque.

(d) Resilience and an "External Load"

Owing to the resilience of shafting in high-power servos, the instantaneous position of the load can be appreciably different from that of the resetting shaft. The load is then said to be "external" to the servo loop. A proper treatment of this subject is outside the scope of this Note (see Ref. 4), but the method of solution will be indicated.

To simulate an external load, two effects of resilience must be considered. These are the displacement between the resetting shaft (θ_o) and the load position (θ_L), and the reflected torque (T_L) at the resetting shaft. It can be shown that the equation relating the load position to the resetting shaft is:-

$$\theta_L/\theta_o = K_L/(I_L p^2 + f_L p + k_L)$$

where I_L, f_L and K_L are respectively the inertia, viscous damping, and stiffness of the load. The equation will be recognised as the same form as equation (1) of subsection 3.2. The reflected torque is given by:-

$$T_L = (\theta_o - \theta_L)K_L$$

A diagram showing the simulated system is shown in Fig. 13(c).

4. APPLICATIONS

4.1 General

The details of some applications of the simulator are briefly described.

It is difficult to describe these experiments without considering the servo problems involved, a task outside the scope of this note. The examples have therefore been selected to demonstrate the use and accuracy of the simulator.

A unity time scale is normally employed. This implies that servo natural frequencies, time delays, velocities, etc., have the same time scale as the "real" system. This is not essential, the time scale used can be arranged to suit the problem, provided that consistency is maintained in all the parameters.

4.2 Use of Linear Transfer Functions

The transfer functions (1) and (2) of subsection 3.2 were set up on the simulator using the method of Fig. 12(a) and (b). In one particular experiment the transfer functions were:-

$9/(p^2+3p+9)$ and $(4p+4)/(p^2+4p+4)$ respectively.

The components used were:-

$R = 1$ Megohm	$R = 1$ Megohm
$R_3 = 1$ Megohm	$R_1 = R_2 = 0.5$ Megohm
$C_1 = C_2 = 1$ microfd	$C_1 = C_2 = 1$ microfd
$R_1 = R_2 = 0.33$ Megohm	$R_3 = 1$ Megohm

When tested with a 10 volt input step, the output response of each loop did not deviate by more than 2% from the theoretical value.

In a specific problem these two servos, operating in cascade, had two possible inputs; a step-function of constant velocity or a step-function of position having an exponential rise (i.e. $1-e^{-kt}$). It was required to know the magnitude of the error between input and the final output and the manner in which this varied for changes in the parameters of the servo system. To have calculated the error would have been very laborious. The actual figures are of no general interest but the parameters selected for the final electromechanical system gave results within $\pm$ 3% of those forecast by the simulator.

4.3 Simple Non-Linearities

(a) Delays

It can be shown that if a servo, described by equation (2), be connected with two simple exponential delays in the resetting loop, the system will oscillate if

$$T_1 + T_2 + \frac{4T_1T_2}{T_1+T_2} = \frac{2}{b}$$

where T_1 and T_2 are the values of the two delays and $a = b$ in equation (2).

The frequency of oscillation will be

$$\omega = \frac{2b}{T_1+T_2}$$

As a check on the accuracy of the simulator, delays were introduced in the loop and their value increased until oscillations were just maintained. The results obtained are compared with calculated figures:-

	Measured	Calculated	Error	Measured	Calculated	Error
T_1	0.55	0.546	0.7%	0.14	0.14	-
T_2	0	0	-	0.145	0.14	3.5%
ω	3.59	3.66	1.7%	4.87	4.77	2.1%

As an example nearer a practical case, Fig. 14(a) shows the effect on transient behaviour of delays of 0.1 sec. and 0.2 sec. in the resetting loop of a servo where a = b = 2. Figure 14(b) shows the effect when the delay occurs in the path of the error signal. These results are traced from pen-recordings.

(b) Backlash

Figure 14(c) shows the effect of backlash between motor and resetter. The servo simulated had a transient response given by equation (1) where a = 0.5 and b = 2. The input was a 10 volt step and the backlash was set up for 1 volt. This simulates the effect of backlash between motor and resetter, of the order of one degree, for a velocity feed-back stabilised servo. It shows the "backlash oscillation" effect when motor friction damping is negligible.

(c) Velocity Limiting

An investigation was made into the additional smoothing achieved from a servo whose output velocity was deliberately limited. A signal from the sine-wave generator was fed to the transfer function of equation (2) with a = b = 2. The input amplitude was 5 volts and the frequency varied from 0.05 c.p.s. to 2.0 c.p.s. The output amplitude was measured with no limiting and then with limiting set at 5.3 volts/sec. and 3.0 volts/sec. (i.e. the voltage limit to the grid of the last integrator was 2.65 volts and 1.5 volts). A cross-check on the accuracy of the limit voltage can be obtained by actual measurement of the maximum output velocity from pen recordings.

The theoretical curves are shown in Fig. 15, with the measured values shown as X. The maximum error is 5% and the average error is 2%.

An input from the noise source was later used in this experiment.

4.4 Simulation of Electro-mechanical Systems

To clarify the arithmetic involved in the simulation of an electro-mechanical system, consider the following example. A large servo-motor is driven by a D.C. amplifier from magslip error signals. The system is to be stabilised by a differentiated tacho-generator signal. The following parameters are known:-

(i) Motor torque - 40 oz in. maximum

(ii) Inertia at motor shaft - 10 oz in.

(iii) Viscous damping at motor shaft - 0.2 oz in./rad./sec.

(iv) Starting friction at motor shaft - 10 oz in. (Assumed constant with speed)

(v) Gear ratio between motor and load - 360:1

(vi) Scale of resetting signal - 2 volts/1° misalignment.

(vii) Amplifier/motor constant (kG) such that full torque is produced by the motor for a misalignment equal to 0.5 degrees.

(viii) Backlash between motor and load equal to 0.5 rev. of the motor.

(ix) Tacho-generator produces 2 volts/r.p.s.

(x) Smoothing time constant of error circuit - 0.1 sec.

The system is shown in Fig. 16, and the procedure adopted was as follows:-

(a) Blocks (1) and (2) represent the error detecting circuit of the system and include the smoothing time constant. The voltage at point B represents torque and this must be limited to 40 oz in. An arbitrary scale of 1 volt = 10 oz in. was chosen for this point and the limit block (10) set to 4 volts.

(b) The scale of voltage representing θ_0 is given by (vi) above as 2 volts/1°. Since there is 360:1 gear ratio between the motor θ_m, and θ_0 the scale of θ_m becomes 2 volts/rev. This is rather low for the backlash simulation, but is used to avoid further scaling. Block (9) is therefore set up for a "backlash" of 1 volt.

(c) The input to the simulated "motor", blocks (5) and (6), is given by (vii) above. Hence point (c) must have a scale 1 volt = 40 oz in.

(d) Since the viscous damping, f, has a small value, the method of 3.4(b) is not suitable. The equation of motion is

$$T = Ip^2\theta_m + fp\theta_m$$

using the same nomenclature as in 3.4(b).

Hence $$\theta = (1 - \frac{f}{T}p\,\theta)/\frac{I}{T}p^2$$

Neglecting viscous damping we have:-

$$\frac{T}{I} = \frac{40}{10} \times \frac{32\times12}{2\pi} \times 2 = 490 \text{ volt/sec}^2$$

Assuming equal integrating time constants, for an input scale of 1 volt = 40 oz in., time constants of 1/22 secs. will give the performance specified.

The torque due to viscous damping is 0.2 oz in./rad./sec. At the output of block (5) this becomes 0.005 volt/rad./sec. With an integrating time constant of 1/22 sec., the voltage at the output of block (5) is $2/(22\times2\pi) = 0.0145$ volt/rad./sec. Hence this voltage multiplied by 1/3 in block (4) is subtracted from the torque voltage and provides the correct viscous damping.

(e) The remainder of the simulation is straightforward. It should be noted that, for stability with the large smoothing delay, it was found necessary to incorporate "phase advance" of the error signal. The circuit of Fig. 17(h) was used. It was assumed that, in the real system, this phase advance would be incorporated with no loss of D.C. gain of the error signal.

The servo performance was adjusted to give a natural frequency of about 2 rad./sec. The response to a position-step input had an overshoot of 15%.

The scaling used in this simulation is not necessarily suitable for all investigations; it was chosen to demonstrate the method involved. In particular, the scaling of the limits and the backlash is as small as practicable.

5. CONCLUSIONS

The simulator described is simple and is easy to construct but has an inherent accuracy sufficient for the investigation of most closed-loop problems. It is a versatile instrument and the inclusion of a low frequency noise source has been a useful development for dealing with servos associated with a radar.

The simulator has been used by the author almost exclusively on combinations of various transfer functions; simple non-linearities have been included. It has facilitated the optimisation of these systems and reduced the labour in calculating the performance of the "real" system. It has been useful in determining the importance of various mechanical parameters as it will indicate a deviation from the simple transfer function ascribed to the system. High orders of accuracy from the component units of the simulator are not necessary for this work. An accuracy of 3% is a reasonable limit.

High power servos, including complex mechanical characteristics, can generally be simulated in some detail. The deleterious effect of backlash and delays in high-order high-natural-frequency systems can be demonstrated; and a useful guide to final performance can be obtained. Since the mechanical properties of the "real" system are seldom known with a high degree of precision, a more accurate simulator would be of doubtful advantage.

In future work on the simulator, it is hoped to investigate some ON-OFF control systems.

6. ACKNOWLEDGMENTS

The author of this technical note wishes to acknowledge the work of Mr. P. F. C. Griffiths on the first simulator and Mr. P. E. H. Pearce for the detailed circuit design of the simulator described in this note.

7. REFERENCES

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2. BURT, E. G. C. LANG, O. H. — "Function Generators Based on Linear Interpolation" R.A.E. Technical Note G.W.244

3. BENJAMIN, R. — "The Representation of Mathematical Functions of a Single Variable by a Succession of Linear Approximations" A.S.R.E. Technical Note NX/50/8.

4. BROWN, G.S. CAMPBELL, D.P. — "Principles of Servomechanisms" John Wiley & Sons, 1948.

APPENDIX

SIMPLE OPERATIONS USING FEEDBACK AMPLIFIERS

1. METHOD

In Fig. 17(a) Z_1 and Z_2 are passive networks of resistances and condensers and A is a high gain D.C. amplifier. Then:-

$$\frac{V_o}{V_g} = -A$$

and, applying Kirchoff's law,

$$\frac{V_i - V_g}{Z_1} = \frac{V_g - V_o}{Z_2}$$

Hence $$\frac{V_o}{V_i} = -\frac{AZ_2}{(A+1)Z_1 + Z_2} \qquad \text{........(1)}$$

and $$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \quad \text{when A is very large ..(2)}$$

Figure 17 summarises the most usual functions.

2. EFFECT OF FINITE GAIN OF D.C. AMPLIFIER

(a) Introduction

As the gain of the D.C. amplifier used in this simulator is finite, it is worth considering the error introduced by using equation (2) instead of equation (1). Two units will be considered: the adder and the integrator.

(b) Addition

Applying the method used to derive equation (1) to the addition circuit shown in Fig. 17(b) we have,

$$V_o = -\frac{A}{1+A} \times \frac{R_2}{R_1} \times \frac{(V_1 + V_2)}{1 + \left(\frac{2R_2}{R_1} \times \frac{1}{1+A}\right)}$$

The simplified performance is described by

$$V_o = -\frac{R_2}{R_1}(V_1 + V_2)$$

The error is therefore equal to

$$\left[1 - \left\{\frac{A}{1+A} \times \frac{1}{1 + \left(\frac{2R_2}{R_1} \times \frac{1}{1+A}\right)}\right\}\right] 100\%$$

If $A = 500$

when $R_2/R_1 = 5$, error = 2%

$R_2/R_1 = 1$, error = 0.4%

$R_2/R_1 = 0.2$, error = 0.1%

These are acceptable errors. However, when the adder is used in the resetting loop, V_2 is negative and, in the steady state condition, equals V_1. The error in this condition is zero.

(c) Integration

Applying equation (1) to the integrator circuit of Fig. 17(c) we have,

$$\frac{V_o}{V_i} = -\frac{A}{(A+1)pT+1} \quad \text{where } T = R_1C_1$$

As a function of time,

$$V_o = -V_iA\left[1 - \exp\left\{-\frac{t}{(A+1)T}\right\}\right]$$

By the simplified theory

$$V_o = -V_i\frac{t}{T}$$

If V_i is maintained constant after t seconds the error is equal to

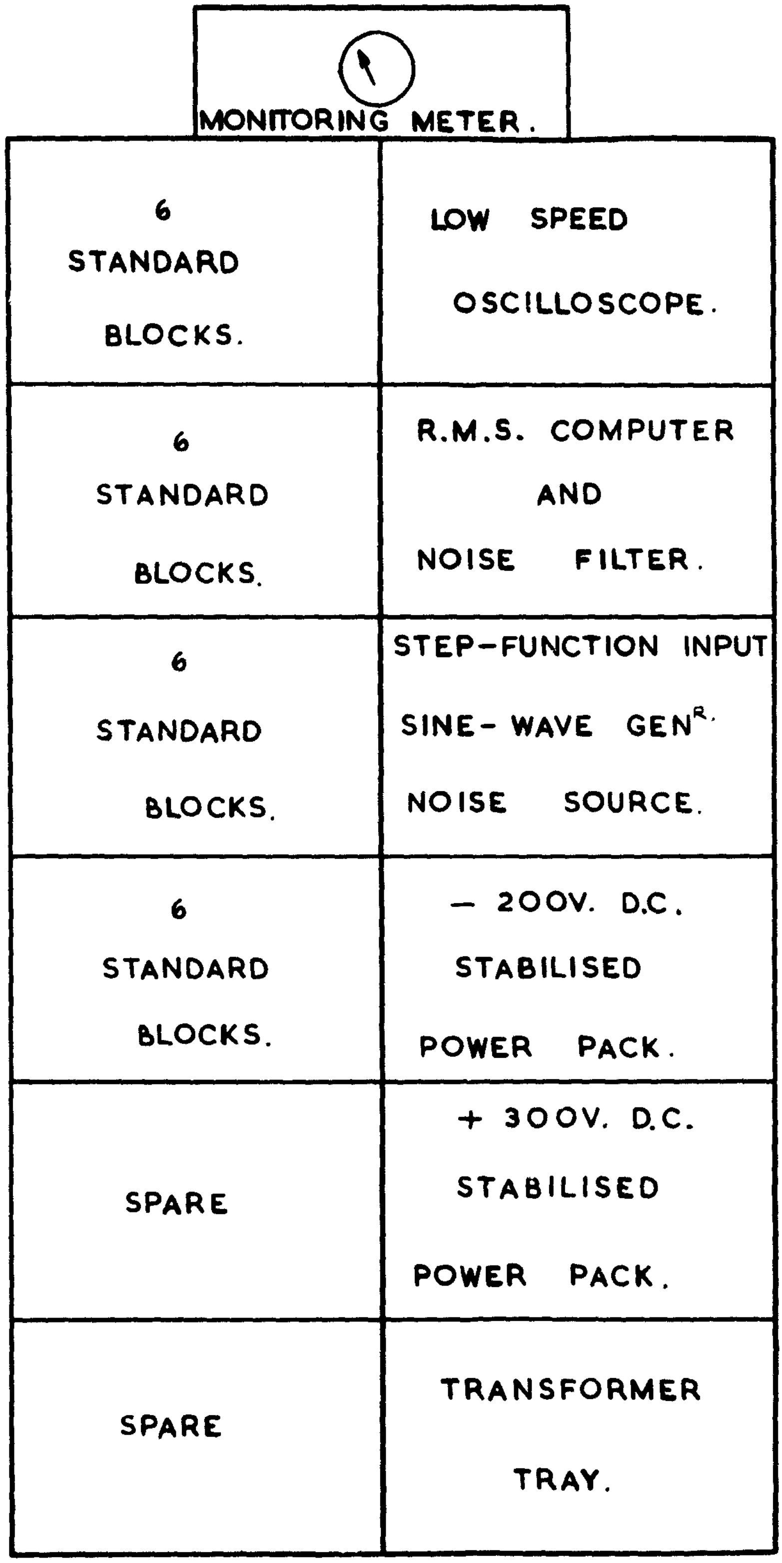
$$\left[1 - \frac{AT}{t}\left\{1 - \exp\left(-\frac{t}{(A+1)T}\right)\right\}\right]\times 100\%$$

The following table indicates the effect of amplifier gain G and the ratio t/T on this error.

	A = 1000		A = 500		A = 250		A = 100	
t/T	2.5	50	2.5	50	2.5	50	2.5	50
% error	0.13	2.46	0.25	4.84	0.5	9.37	1.25	21.3

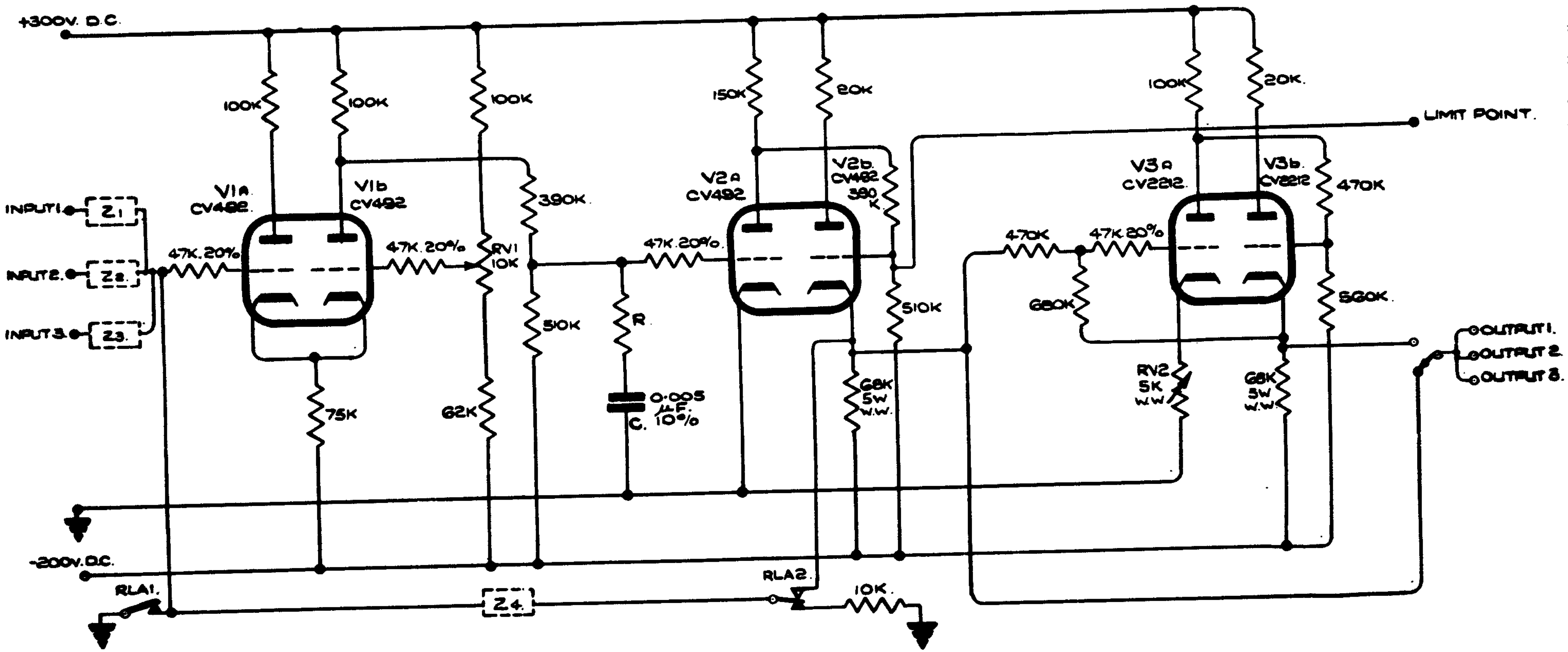
It is emphasised that these are percentage errors for a fixed input. Consideration of a simulated closed loop with step-function inputs will produce much smaller errors. This is because the input to the integrators is, after a short time, reduced to zero. Consideration of the zero-velocity lag servo described in subsection 3.3 of this technical note shows that the maximum error due to the gain of the amplifiers being 500, and not infinity, is less than 0.4%. This is negligible compared with component tolerances and the initial assumptions used in deriving the transfer function.

FIG. 1.



SIMULATOR LAY-OUT.

A.S.R.E. DATE 18-2 TR SRS CH APP. DWG. No. GX992[illegible]



R SELECTED TO STABILISE CIRCUIT NOMINAL VALUE 4·7K.

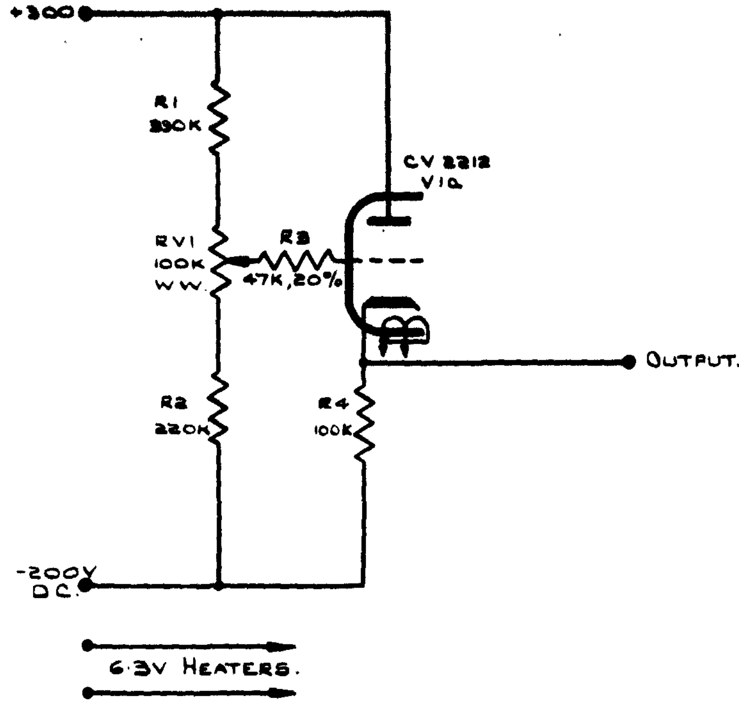
ALL RESISTORS 5% H.S. UNLESS OTHERWISE STATED.

STANDARD D.C. AMPLIFIER.

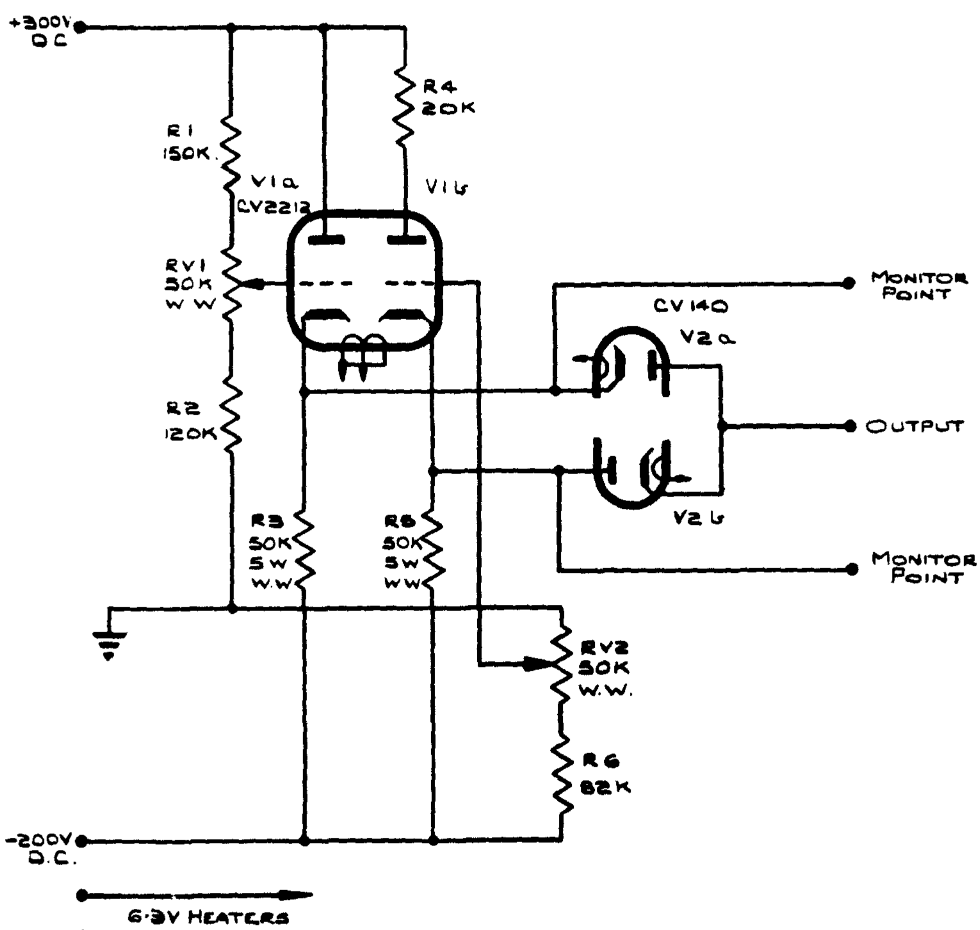
FIG. 2.

A.S.R.E. DATE 1·3·56 TR. B.W. CH. APP. DWG. No. GX99282

FIGS. 3 & 4

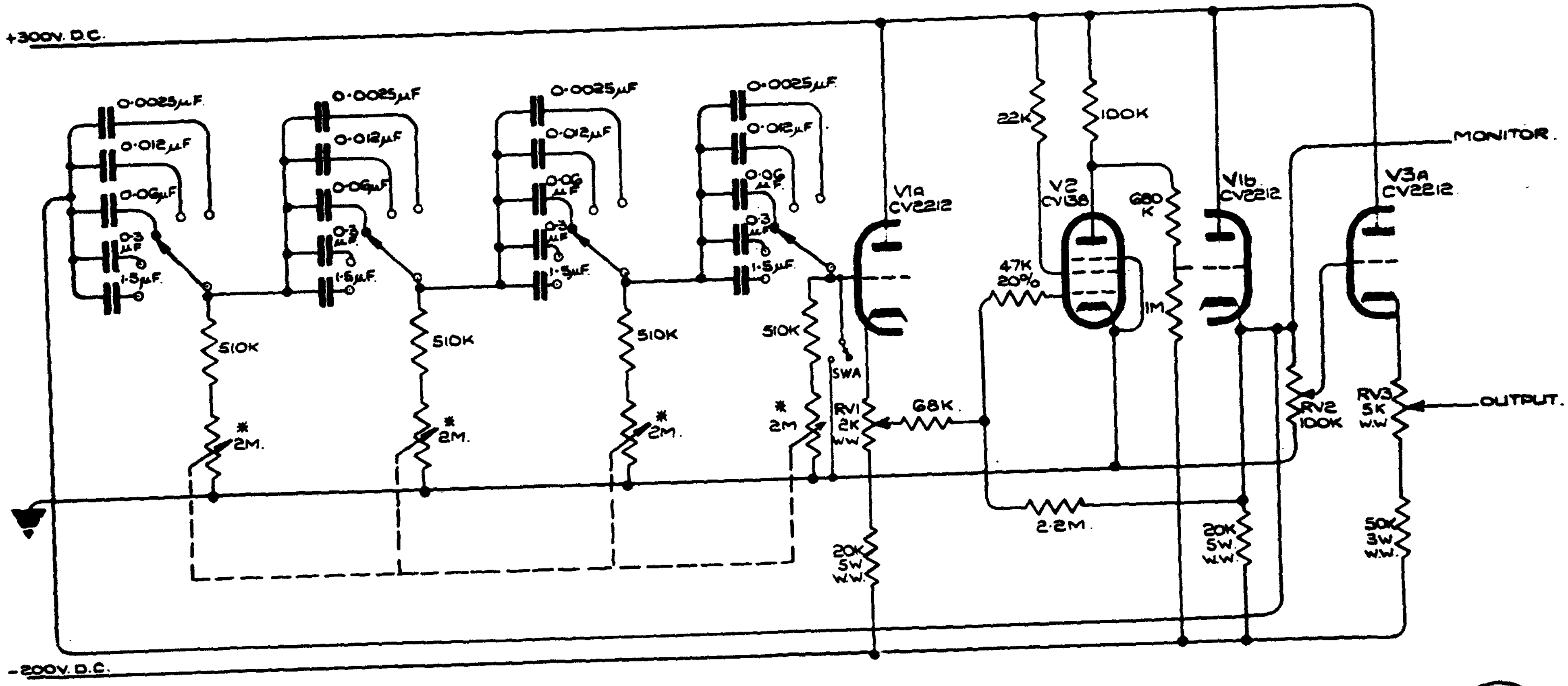


STEP FUNCTION GENERATOR.



LIMIT CIRCUIT.

A.S.R.E. DATE 24.2.55 TR. S&S CH. APP. DWG. No. GX9928



ALL RESISTORS 5% H.S. UNLESS OTHERWISE STATED.
ALL CONDENSERS SELECTED TO 1%.
* ALL THESE POTENTIOMETERS ARE ACCURATELY GANGED.

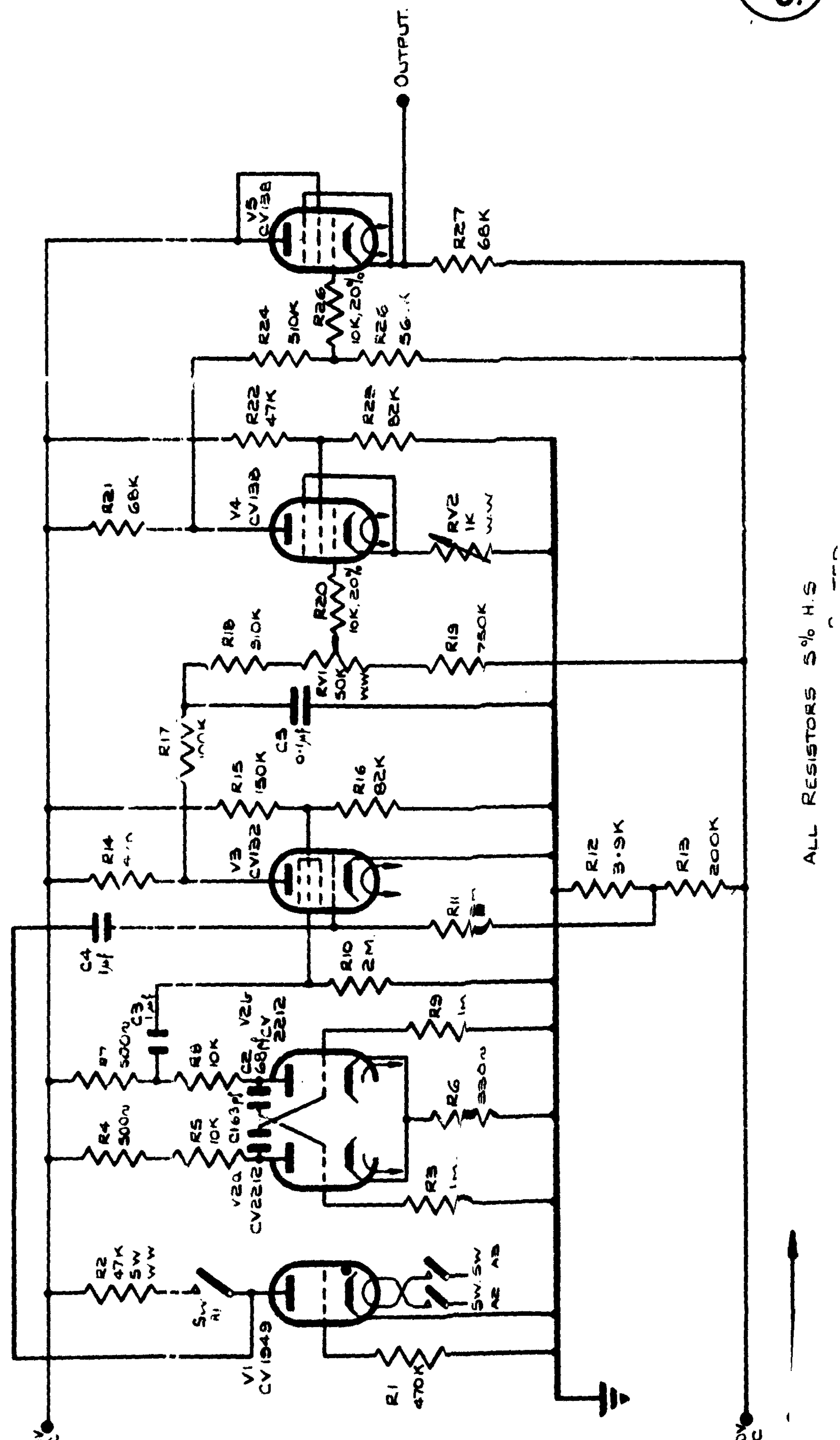
SINE-WAVE GENERATOR.

FIG 5.

A.S.R.E DATE 1.3.55 TR.BW. CH. APP. DWG. No.GX992E

NOISE SOURCE.

FIG. 6.



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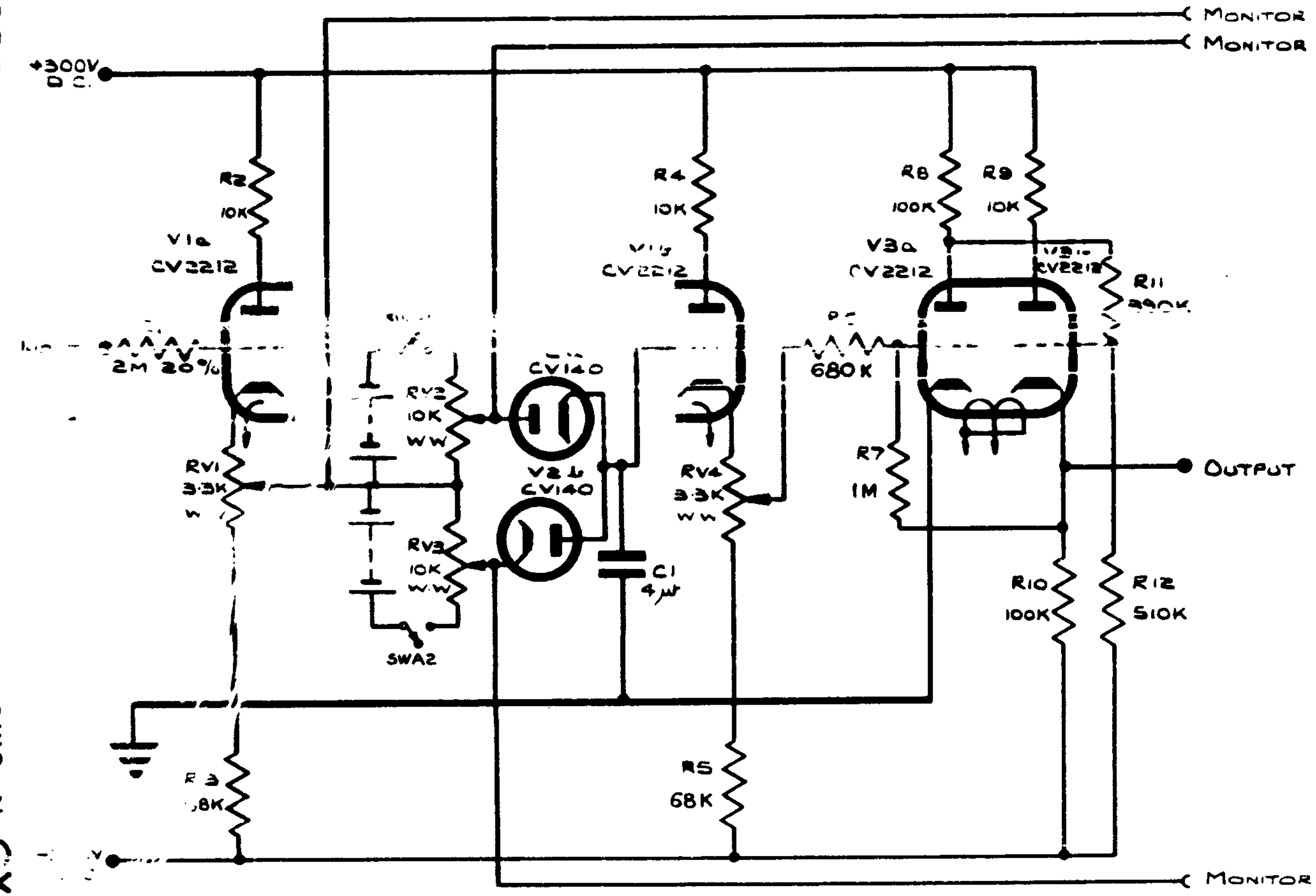
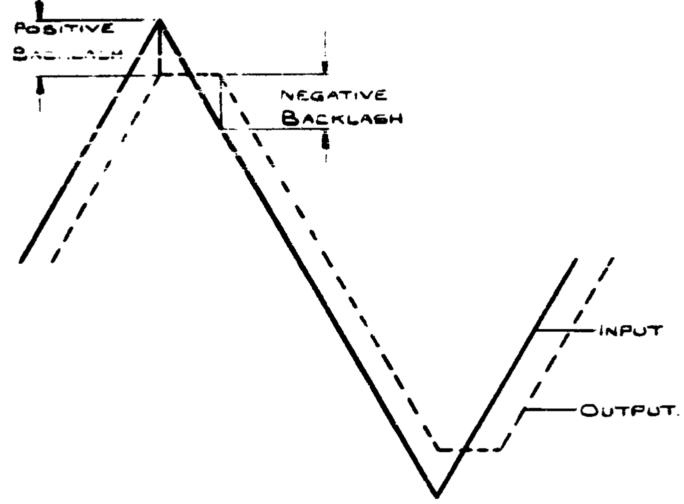


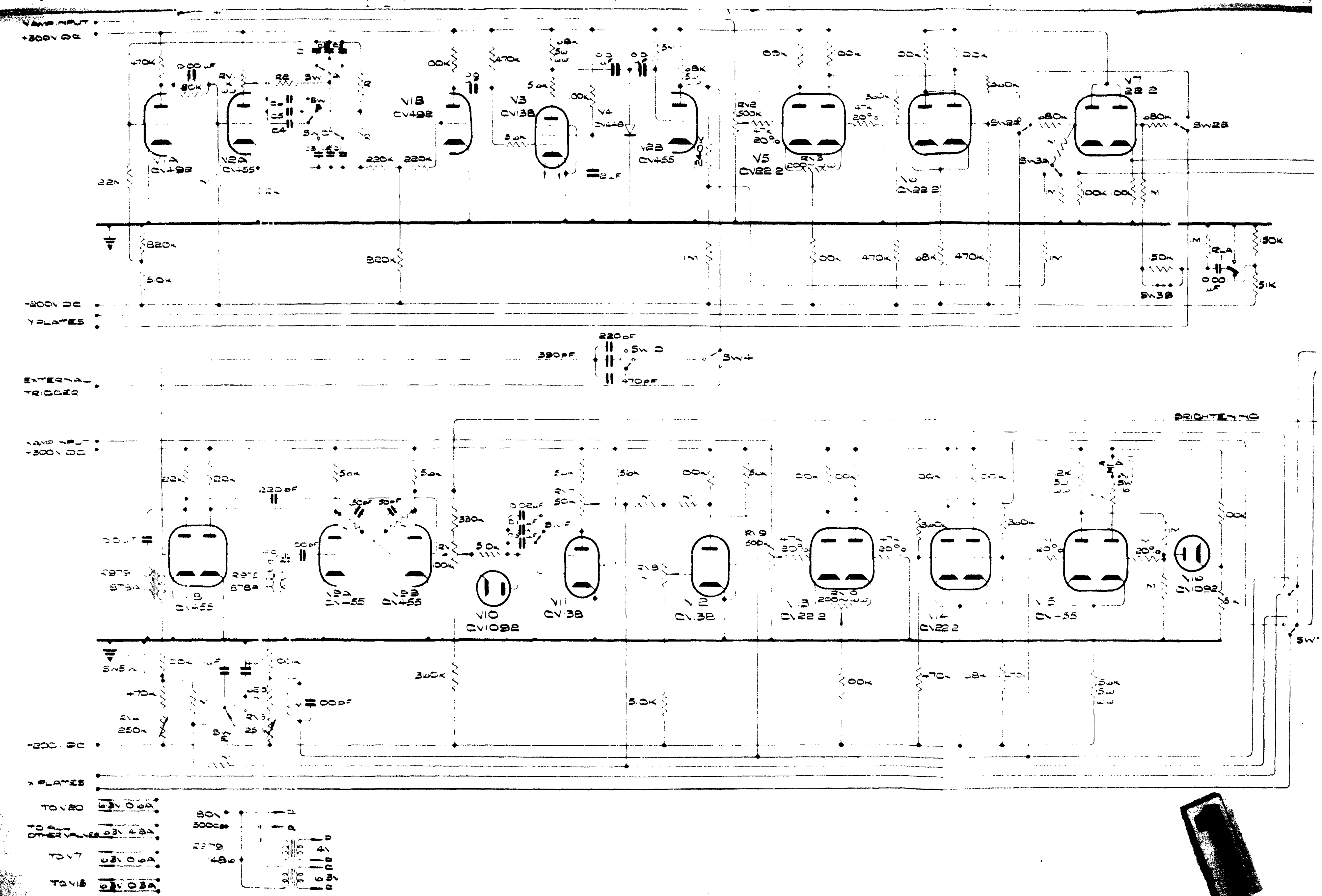
BACKLASH CIRCUIT



CHARACTERISTICS.

FIGS. 7 AND 8

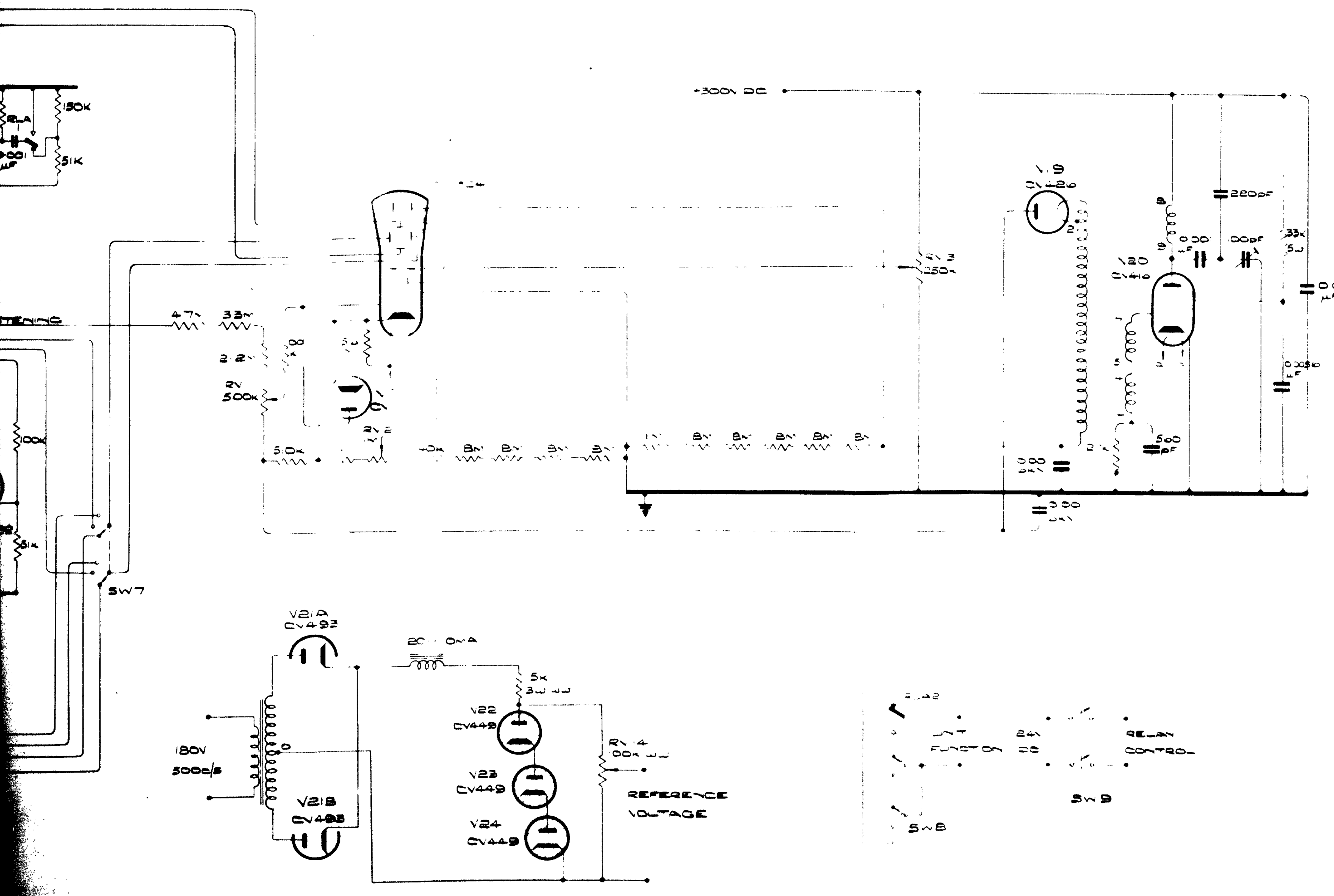
A.S.R.E. DATE 25.2.55 CH. APP. DWG. No. GX99286



A.S.R.E. DATE 3-4-55 TR & B CH APP

G 9-

TED OSCILLOSCOPE



C1 = 0·25μF 1%
C2 = 0·085μF 1%
C3 = 0·0025μF 1%
C4 = 0·5μF 1%
C5 = 0·05μF 1%
C6 = 0·005μF 1%
R1 = 640K 1%
R2 = 320K 1%
ALL OTHER RESISTORS UNLESS OTHERWISE STATED TO BE 5% OHMS

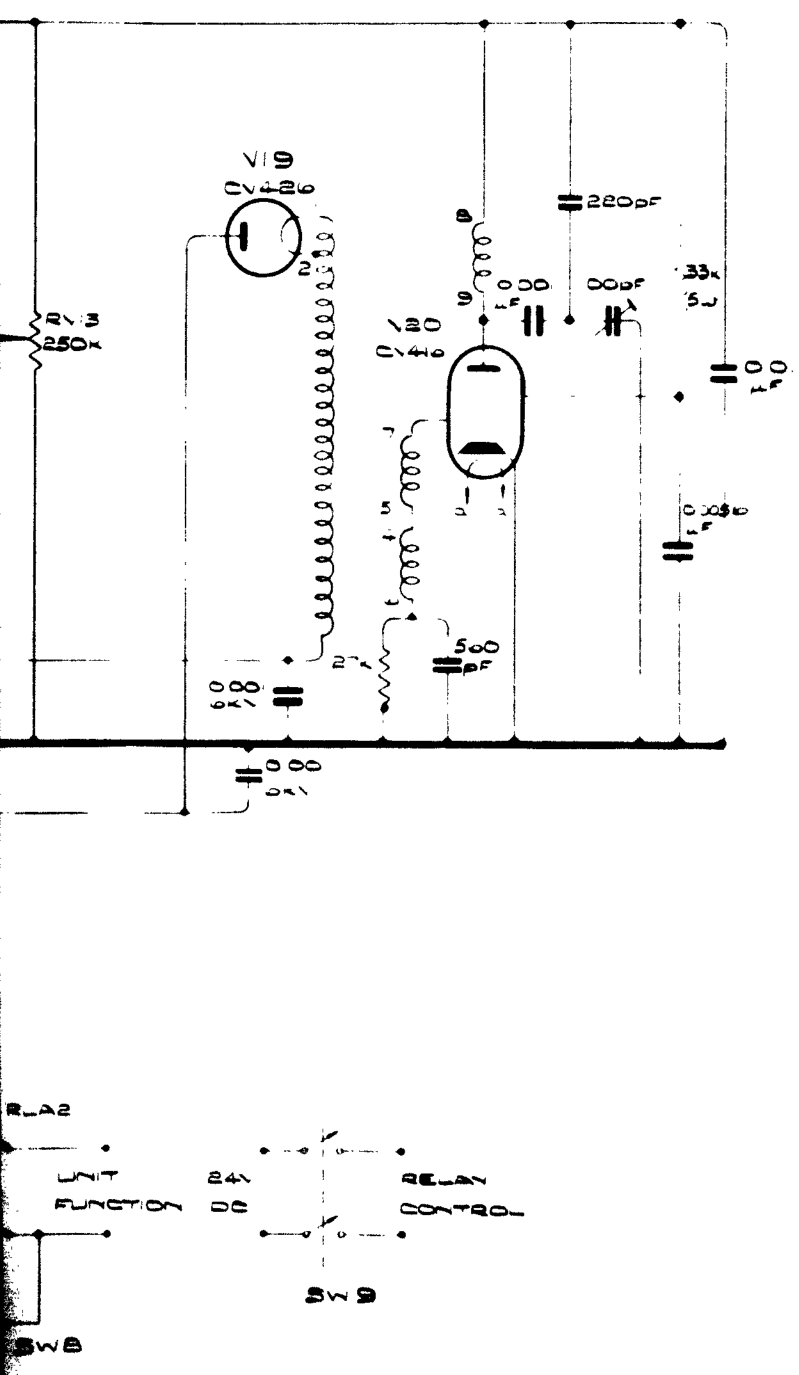
SW1 6 POLE 3 WAY
SW2 2 POLE 2 WAY
SW3 2 POLE 2 WAY
SW4 POLE 1 WAY
SW5 PUSH BUTTON
SW6 POLE WAY
SW7 2 POLE 3 WAY
SW8 2 POLE 2 WAY
SW9 2 POLE 1 WAY

RLA 5000Ω

2

DRG. No. GX 99287

FIG 9 | 9A

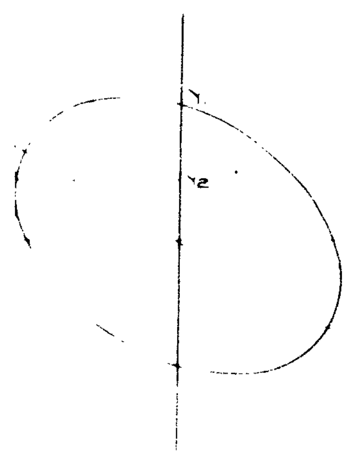


C1 = 0·25µF 1%
C2 = 0·085µF 1%
C3 = 0·0025µF 1%
C4 = 0·5µF 1%
C5 = 0·05µF 1%
C6 = 0·005µF 1%
R1 = 640K 1%
R2 = 320K 1%
ALL OTHER RESISTORS UNLESS OTHERWISE STATED TO BE 5%

SW1 4 POLE 3 WAY
SW2 2 POLE 2 WAY
SW3 2 POLE 2 WAY
SW4 POLE WAY
SW5 PUSH BUTTON
SW6 POLE WAY
SW7 2 POLE 3 WAY
SW8 2 POLE 2 WAY
SW9 2 POLE WAY

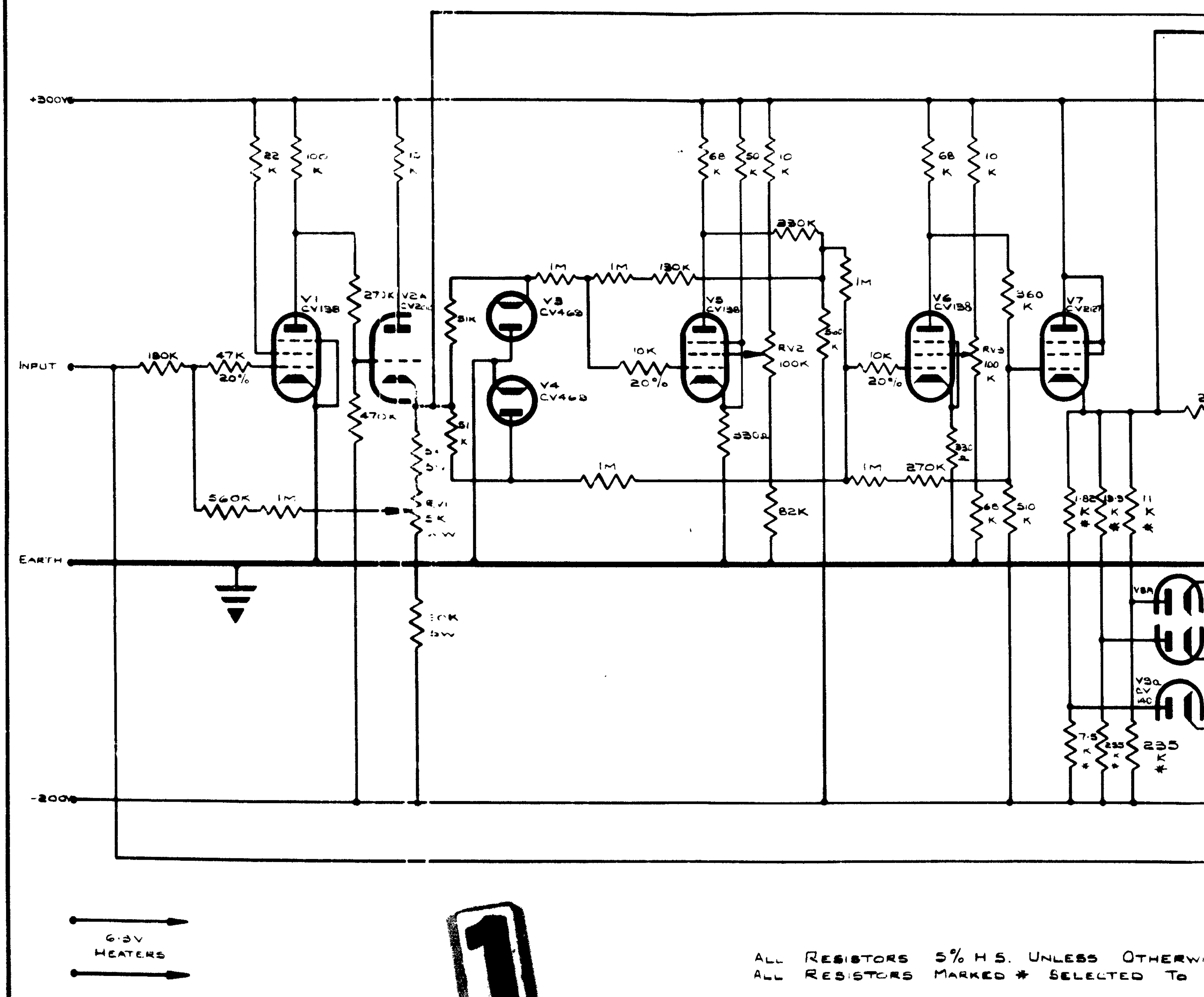
RLA 5000Ω

FIG 9a



3

DRG. No GX 99287

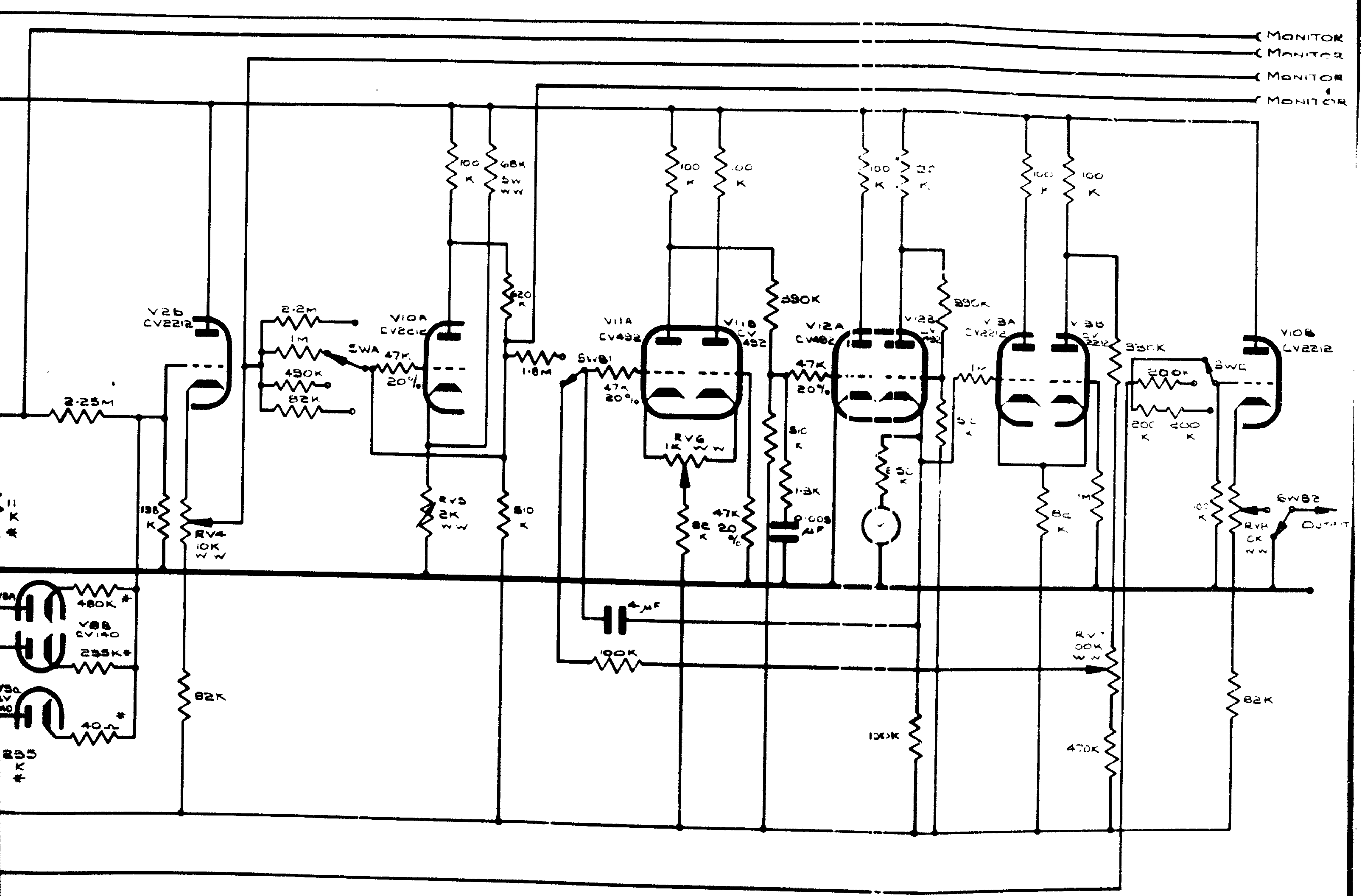


1

All Resistors 5% H.S. Unless Otherwis
All Resistors Marked * Selected To !

Admiralty Signal & Radar Establishment   Date 9-5-55 Tr 505   Ch   App

FIG. 10



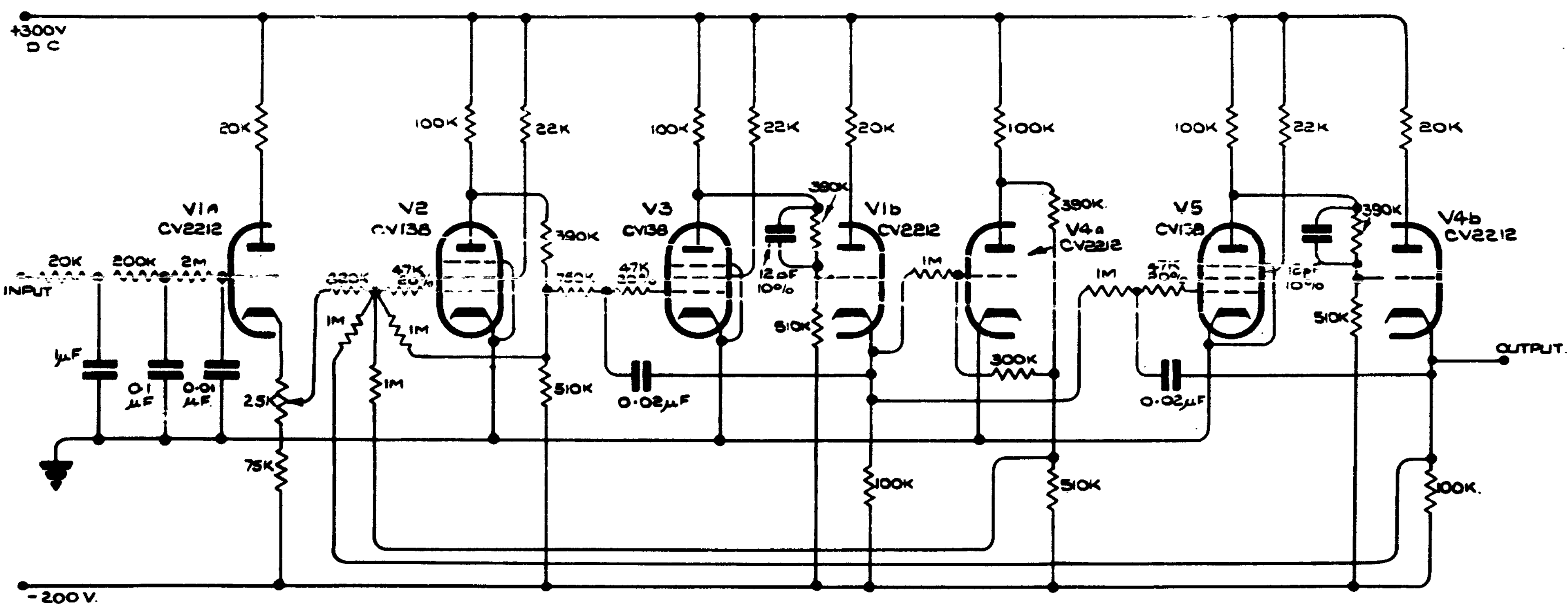
…HERWISE STATED.
To 1%.

SWA - 1 POLE 4 WAY
SWB - 2 POLE 2 WAY TOGGLE.
SWC - 1 POLE 3 WAY

R.M.S. COMPUTER.

12

DRG. No. GX 99288.



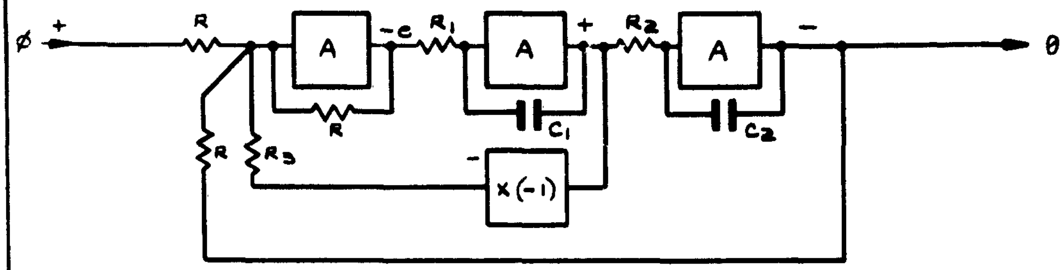
ALL RESISTORS 5% H S UNLESS OTHERWISE STATED
ALL CONDENSERS 5% UNLESS OTHERWISE STATED

NOISE FILTER.

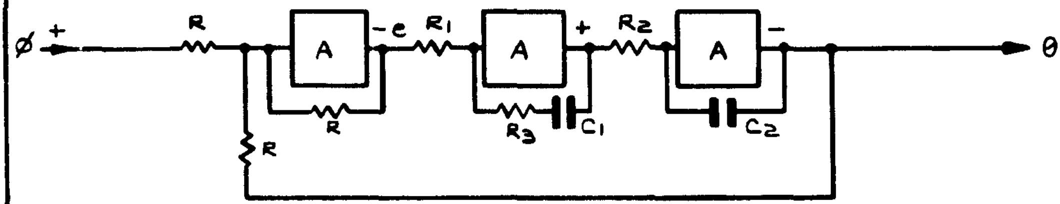
FIG. 11

A.S.R.E. DATE 24 2 55 TR. B.W. CH. APP. DWG. No. GX99290

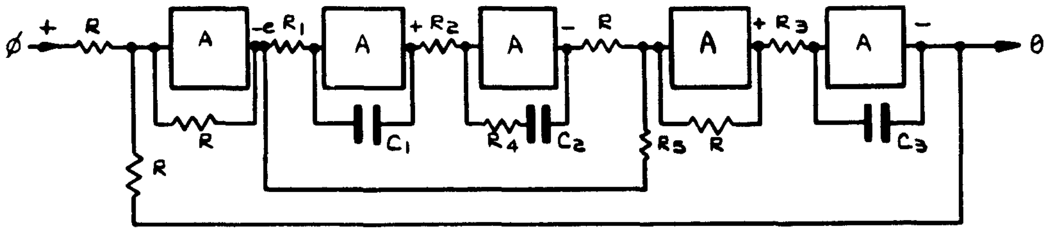
FIG. 12



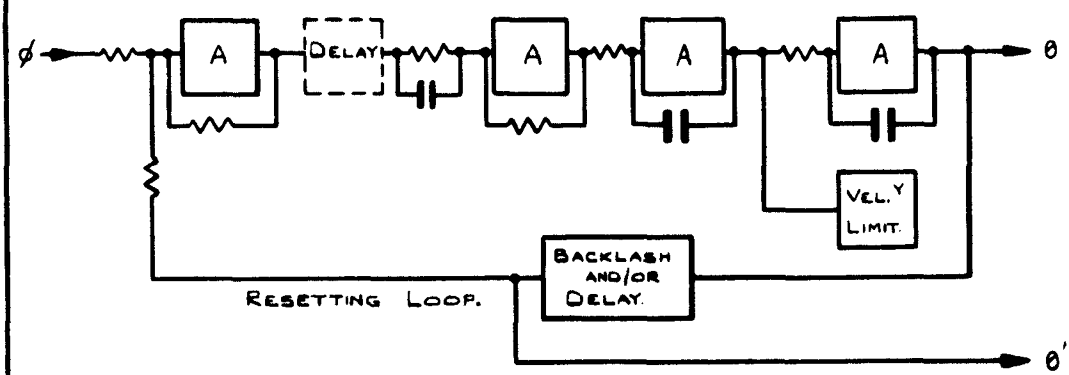
(a) FINITE VELOCITY - LAG.



(b) ZERO VELOCITY - LAG.



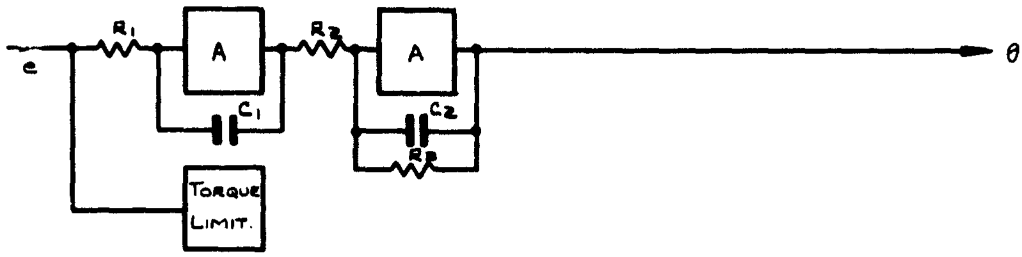
(c) ZERO ACCELERATION - LAG



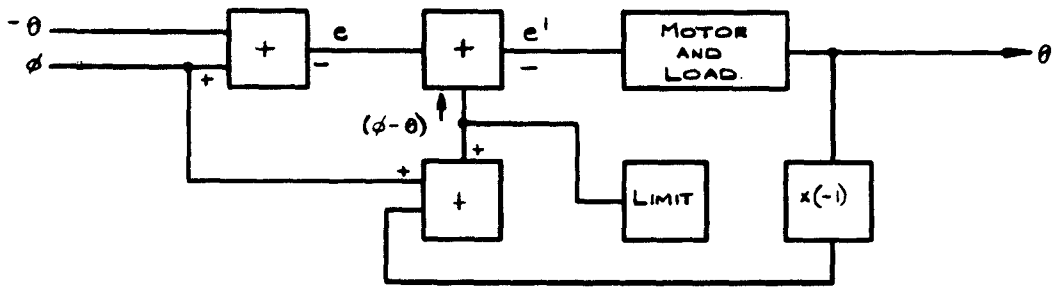
(d) SIMPLE NON LINEARITIES IN A "PHASE ADVANCE" STABILISED SERVO.

SIMULATION OF TRANSFER FUNCTIONS.

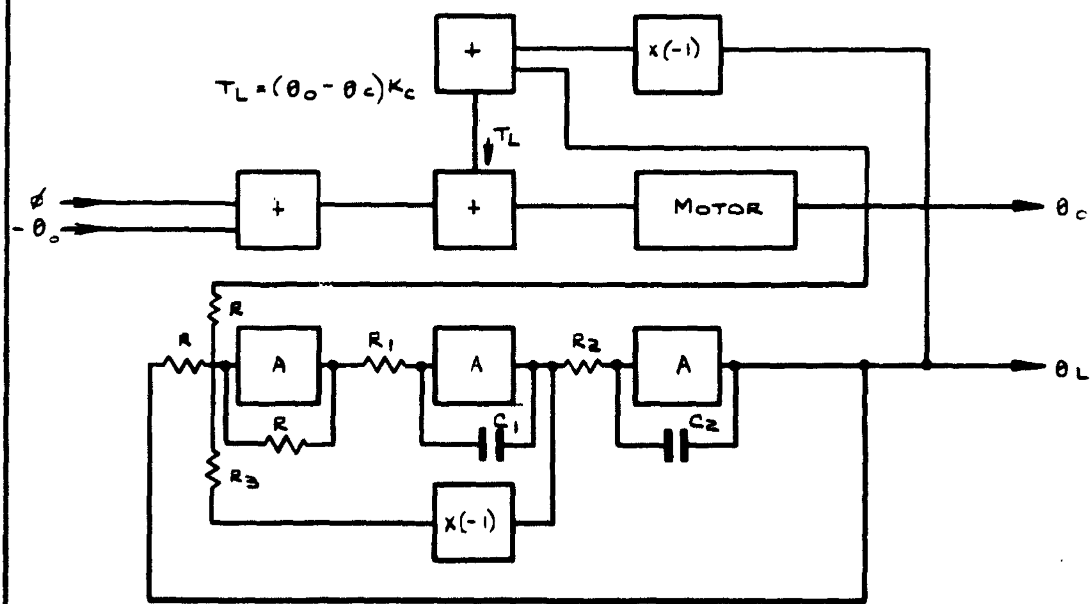
FIG. 13



(a) MOTOR AND LOAD.



(b) STICTION



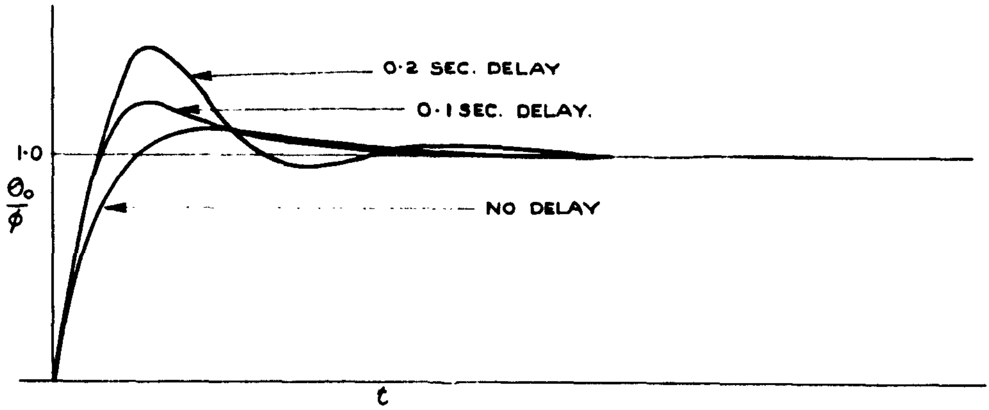
$$\text{WHERE} \quad \frac{\theta_L}{\theta_o} = \frac{K_L/I_L}{p^2 + \frac{f_L}{I_L} + \frac{K_L}{I_L}} = \frac{1/R_1C_1R_2C_2}{p^2 + pR/R_3R_1C_1 + 1/R_1C_1R_2C_2}$$

(c) EXTERNAL LOAD.

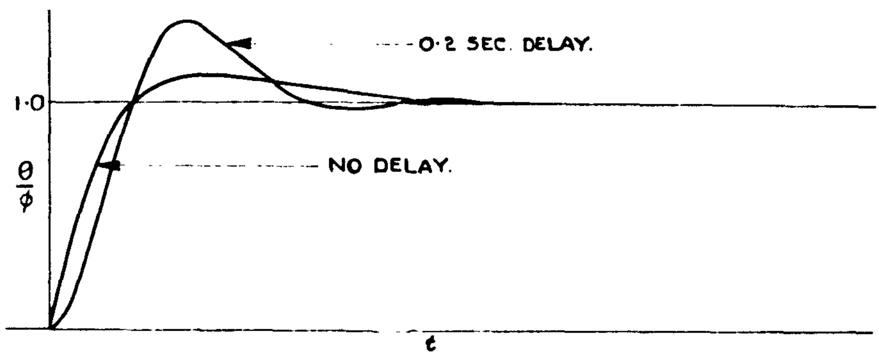
SIMULATION OF MECHANICAL SYSTEMS.

A.S.R.E. DATE 22·2·55 TR. SDS. CH. APP. DWG. No. GX 99292.

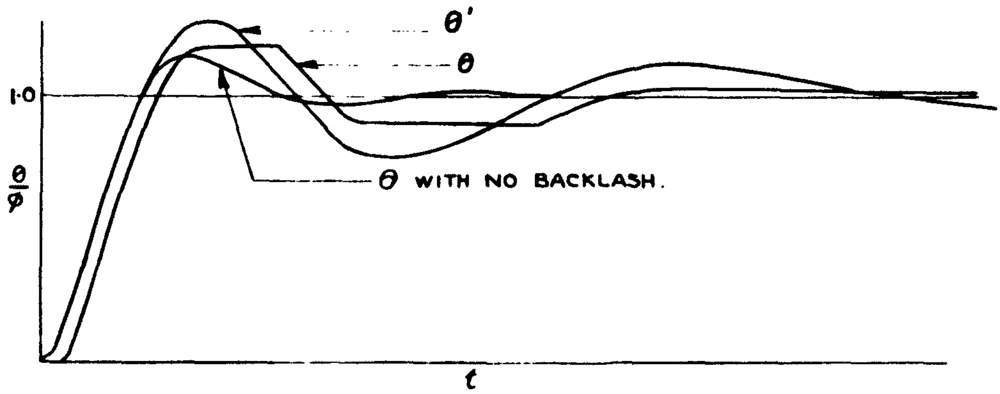
FIG. 14



(a) DELAY IN RESET LOOP.



(b) DELAY IN ERROR LOOP.



(c) BACKLASH IN RESET LOOP.

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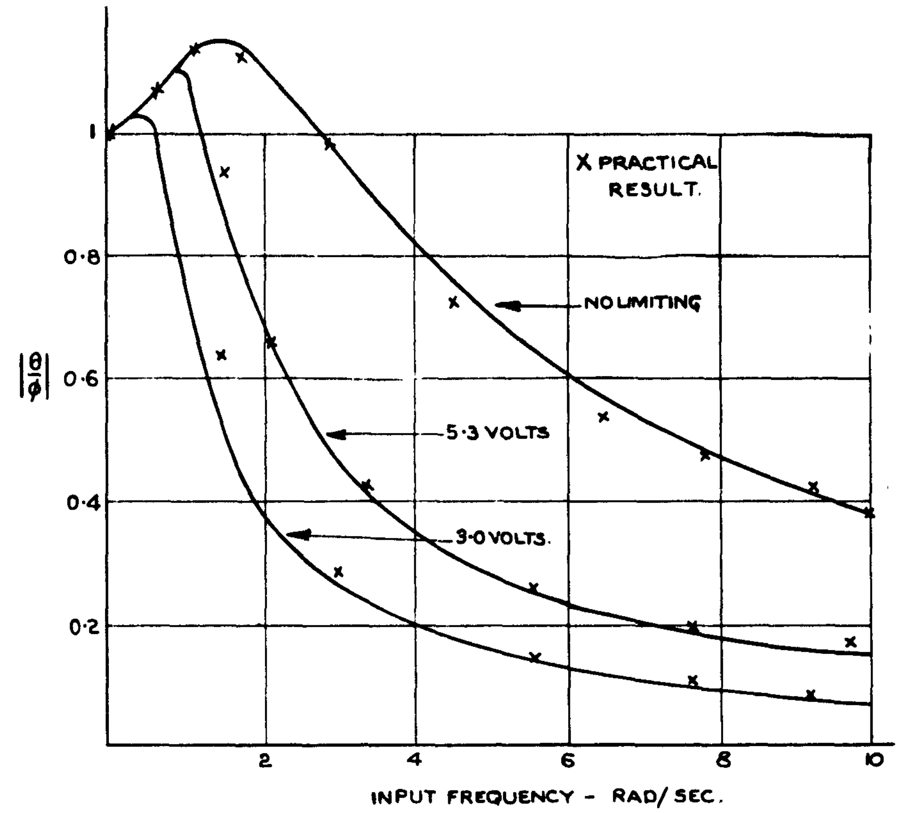
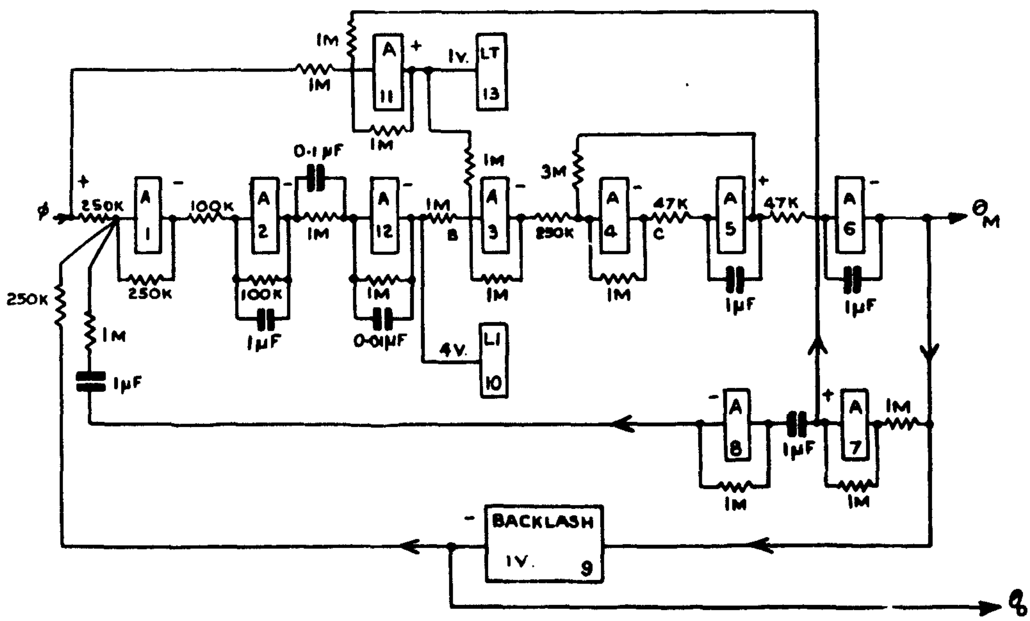
FIGS. 15 / 16

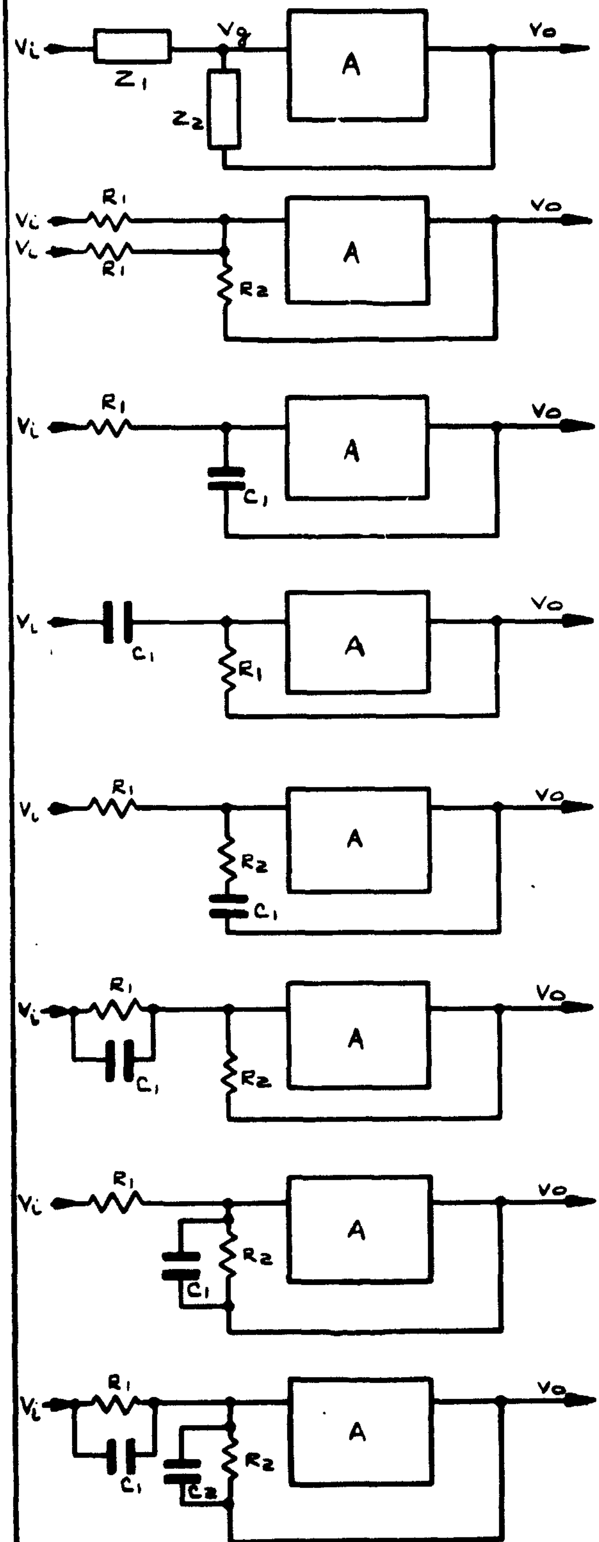


VELOCITY LIMITING.



MECHANICAL SYSTEM.

FIG. 17



$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \qquad (a)$$

$$V_o = \frac{-R_2\,(V_i + V_i')}{R_1} \qquad (b)$$

ADDITION.

$$\frac{V_o}{V_i} = -\frac{1}{p\,C_1 R_1} \qquad (c)$$

INTEGRATION.

$$\frac{V_o}{V_i} = -\,p\,C_1 R_1 \qquad (d)$$

DIFFERENTIATION.

$$\frac{V_o}{V_i} = -\left(\frac{R_2}{R_1} + \frac{1}{p\,C_1 R_1}\right) \qquad (e)$$

AMOUNT + INTEGRAL

$$\frac{V_o}{V_i} = -\left(\frac{R_2}{R_1} + p\,C_1 R_2\right) \qquad (f)$$

AMOUNT + DERIVATIVE

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\left(\frac{1}{1 + p\,C_1 R_2}\right) \qquad (g)$$

SIMPLE DELAY.

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}\left(\frac{1 + p\,C_1 R_1}{1 + p\,C_2 R_2}\right) \qquad (h)$$

"PHASE ADVANCE."

STANDARD FUNCTIONS.

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Date of Search: 29 May 2008

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Former reference (Department) Technical Note GX Series 1
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